

# Shear viscosity of hadronic gas mixtures

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## Abstract.

We investigate the effects of baryon chemical potential  $\mu$  on the shear viscosity coefficient  $\eta$  and the viscosity to entropy density ratio  $\eta/s$  of a pion-nucleon gas mixture. We find that  $\eta$  is an increasing function of  $T$  and  $\mu$ , while the ratio  $\eta/s$  turns to a decreasing function in a wide region of  $T$ - $\mu$  plane. In the kinematical region we studied, the smallest value of  $\eta/s$  is about 0.3.

## 1. Introduction

Small values of the shear viscosity of a hot QCD matter inferred from the RHIC data have lead to a new concept, “strongly-interacting QGP (sQGP)”. It is thus quite important to understand how such small values of the viscosity are realized in the QCD matter. At present, there are *two conjectures* about the behavior of the viscosity. From the analysis of the AdS/CFT correspondence, it has been conjectured that *there would be a lower bound in the “shear viscosity coefficient to the entropy density ratio”*  $\eta/s \geq 1/4\pi$  [1]. The lowest value  $\eta/s = 1/4\pi$  (“the KSS bound”) is satisfied by several super Yang-Mills theories in the large  $N_c$  limit (strong coupling limit), which suggests that the bound could be universal. The second conjecture which is expected to universally hold is based on an empirical observation seen in many substances: *The ratio  $\eta/s$  will have a minimum at or near the critical temperature* [2] (see also [3]). More precisely, the ratio shows a gap at  $T_c$  for the first order transition, while it has a convex shape for the crossover with its bottom around the (pseudo) critical temperature. Recall that the phase transition in QCD is most probably crossover at least for low densities. Therefore, what we naturally expect is that *the ratio  $\eta/s$  in QCD will have the minimum at  $T \sim T_c$ , and the numerical value at that point will be close to the KSS bound  $\eta/s \sim 0.1$ .*

These considerations motivated us to investigate the shear viscosity in QCD *from the hadronic phase*  $T \lesssim T_c$ . Notice that we can indirectly study the properties of sQGP from below  $T_c$  because physical quantities such as the ratio  $\eta/s$  will be continuous at  $T_c$  for the crossover transition. Moreover, inclusion of nucleon degrees of freedom enables us to investigate the dependence of  $\eta/s$  on the baryon chemical potential  $\mu$  and thus to study the behavior of  $\eta/s$  in a wide region of the phase diagram. In this proceedings, we only give the outline of our analyses and show a few numerical results. More details and comparison with the results in literature are available in Ref. [4].

## 2. Theoretical framework: relativistic quantum Boltzmann equations

We compute the shear viscosity coefficient  $\eta$  of a pion-nucleon gas mixture by solving relativistic Boltzmann equations which contain binary scatterings in the collision terms:

$$\frac{1}{E_p^\pi} p^\mu \partial_\mu f^\pi(x, p) = \mathcal{C}^{\pi\pi}[f^\pi, f^\pi] + \mathcal{C}^{\pi N}[f^\pi, f^N], \quad (1)$$

$$\frac{1}{E_p^N} p^\mu \partial_\mu f^N(x, p) = \mathcal{C}^{NN}[f^N, f^N] + \mathcal{C}^{N\pi}[f^N, f^\pi], \quad (2)$$

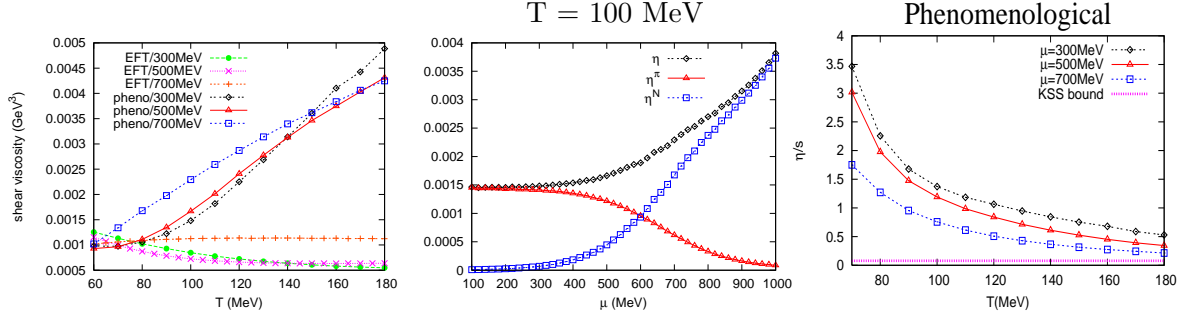
where  $f^{\pi,N}(x, p)$  is the (iso-spin averaged) one-particle distribution of pions or nucleons,  $E_p^{\pi,N} = \sqrt{m_{\pi,N}^2 + p^2}$  and  $\mathcal{C}^{ij}$  is the collision term representing binary ( $2 \rightarrow 2$ ) scattering between particles  $i$  and  $j$  with the effects of statistics included. Also included in the collision terms are the scattering amplitudes, for which we adopt *phenomenological amplitudes* fitted to the experimental data of elastic scatterings in the vacuum. Fit was performed up to scattering energy  $\sqrt{s} = 1.15$  GeV, 2.00 GeV, and 2.04 GeV for the  $\pi\pi$ ,  $\pi N$ , and  $NN$  scatterings, respectively [4]. Note that the phenomenological cross sections are largely different from those of the low energy effective theories where  $\rho$ -meson and  $\Delta$  resonances are not included.

In order to solve eqs. (1), (2) which are nonlinear with respect to  $f^\pi(x, p)$  and  $f^N(x, p)$ , we consider small deviation from thermal equilibrium at temperature  $T$  and baryon chemical potential  $\mu$ . Namely, we linearize the equations for small deviations  $\delta f^{\pi,N}$  as defined by  $f^{\pi,N} = f_0^{\pi,N} + \delta f^{\pi,N}$  with thermal distribution  $f_0^{\pi,N}$  (Chapmann-Enskog method). The shear viscosity coefficient  $\eta$  is given as a function of  $f_0^{\pi,N}$  and  $\delta f^{\pi,N}$ . Notice that  $\eta$  can be decomposed into contributions from pions and nucleons:

$$\eta = \eta^\pi + \eta^N, \quad (3)$$

where  $\eta^\pi$  ( $\eta^N$ ) is given by pion (nucleon) distribution alone:  $\eta^\pi = \eta^\pi[f_0^\pi, \delta f^\pi]$  and  $\eta^N = \eta^N[f_0^N, \delta f^N]$ . On the other hand, we compute the entropy density in the thermal equilibrium, thus it depends only on  $f_0^{\pi,N}$ .

Lastly, let us briefly discuss the range of validity of our framework. Our calculation has limitation in two different aspects. First, since we use the phenomenological amplitudes which are fitted to the data up to some finite values of scattering energy, we have to be careful if our results do not contain significant contributions from outside of the fit regions. This may be specified, for example, by a condition  $\langle s \rangle + \Sigma < s_{\max}$  where  $\langle s \rangle$  and  $\Sigma = \sqrt{\langle s^2 \rangle - \langle s \rangle^2}$  are the average scattering energy squared and its standard deviation, and  $s_{\max}$  is the maximum energy squared of the fit. Second, since our framework is based on the Boltzmann equations which are only justified for dilute gases, the density of particles must be small enough. This is achieved when  $\lambda \gg d$  where  $\lambda$  is the mean-free path  $\lambda = 1/n\sigma$  with  $\sigma$  being the cross section, and  $d$  is the interaction range  $d \sim 1/m_\pi$ . Combining these two conditions, we find that our framework should give a reasonable description in a wide region of the hadronic phase on the  $T$ - $\mu$  plane: The boundary is given by (a quarter of) the elliptic curve connecting  $(T, \mu) \sim (130, 0)$  and  $(0, 950)$  in unit of MeV. This is highly contrasted with the low energy effective theories whose range of validity is quite narrow.



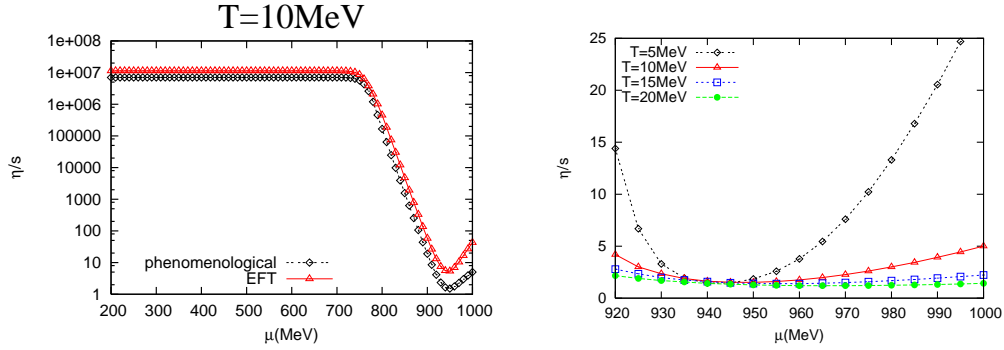
**Figure 1.** Left:  $T$  dependence of  $\eta$  at  $\mu = 300, 500, 700$  MeV (compared with the results of low energy effective theories). Middle:  $\mu$  dependence of  $\eta$  at  $T = 100$  MeV, and its decomposition  $\eta = \eta^\pi + \eta^N$ . Right: The ratio  $\eta/s$  as a function of temperature at  $\mu = 300, 500, 700$  MeV.

### 3. Numerical results: $\mu$ dependence of $\eta$ and $\eta/s$

Fig. 1 shows our numerical results of  $\eta$  and  $\eta/s$  of a pion-nucleon gas mixture. The left panel is the  $T$  dependence of  $\eta$  at three different values of  $\mu$ . In the range of temperature shown here,  $\eta$  is an increasing function of  $T$ . On the other hand, the result of low energy effective theories is a decreasing function of  $T$ , which is however not trustworthy because the upper limit of temperature where the low energy effective theory is valid is below  $T \sim 70$  MeV. If one looks at the window  $80 \text{ MeV} \lesssim T \lesssim 130 \text{ MeV}$ , one finds that  $\eta$  increases with increasing  $\mu$ . Mechanism of increasing  $\eta$  can be understood by the inspection of the middle panel where the  $\mu$  dependence of the total  $\eta$  as well as each contribution  $\eta^\pi$  and  $\eta^N$  is plotted. Since  $\eta$  will be inversely proportional to the cross section, one naively guesses that the inclusion of nucleon degrees of freedom will reduce the viscosity (because the effective cross section will enhance). This is indeed the case for  $\eta^\pi$ . But in fact the contribution of nucleon viscosity itself is large, and thus the total  $\eta$  increases with increasing  $\mu$ . On the other hand, due to rapid growth of entropy density, the ratio  $\eta/s$  turns to a *decreasing* function of  $T$  and  $\mu$  in a wide region on the  $T$ - $\mu$  plane (right panel). In the kinematical region we investigated  $T < 180$  MeV,  $\mu < 1$  GeV, the smallest value of  $\eta/s$  is about 0.3, which is realized at the edge of the validity region:  $T \sim 150$  MeV and  $\mu \sim 940$  MeV. Therefore, *with increasing  $T$  or  $\mu$ , the ratio  $\eta/s$  becomes as small as the conjectured bound  $\eta/s = 1/4\pi \sim 0.1$ , but still keeps above the bound within the region of validity of our framework.* Notice that the smallness of  $\eta/s$  in the hadronic phase and its continuity at  $T \simeq T_c$  (at least for crossover at small  $\mu$ ) implies that the ratio will be small enough in the deconfined phase  $T \gtrsim T_c$ .

### 4. Numerical results: Approaching phase boundary

We do not expect we can accurately describe phase transitions within the framework of (standard) Boltzmann equations which are appropriate only for dilute gases. Still, with the help of the conjectures (in particular, the second conjecture) discussed in Introduction, we can extract some qualitative information about phase transitions from the extrapolation of our results towards critical  $T$  or  $\mu$ . Below, we consider two cases: (i) towards higher  $T$  at zero  $\mu$ , and (ii) towards higher  $\mu$  at low  $T$ .



**Figure 2.** Left: Global structure of  $\mu$  dependence of  $\eta/s$  at low temperature  $T = 10$  MeV. Right: The valley structure of  $\eta/s$  around  $\mu = 950$  MeV changes depending on the temperature  $T = 5, 10, 15, 20$  MeV.

#### 4.1. Towards higher temperatures – chiral phase transition

The second conjecture tells us that  $\eta/s$  will have a characteristic structure around the phase transition point: it has the minimum at the critical points. This implies that, in the hadronic phase, we will see the left hand side of the valley. This is consistent with the numerical results as shown in the right panel of Fig. 1. If this is true, then one can guess the position of the critical temperature from the curve of  $\eta/s$  in the hadronic phase. For example, we can approximate the curve by a quadratic function of  $(T - T_0)$  with  $T_0$  being the reference temperature. Then, the critical temperature may be identified with the temperature where the slope of the curve is zero. We have done this for a pion gas ( $\mu = 0$ ) and for  $T_0 = 140$  MeV, and obtained a reasonable value  $T_c \simeq 173$  MeV.

#### 4.2. Towards higher densities – nuclear liquid-gas transition

At relatively high temperature  $T \sim 100$  MeV, the ratio is a monotonically decreasing function of  $\mu$ . However, there emerges a nontrivial structure at low temperature and at around normal nuclear density, as shown in the left panel of Fig. 2. According to the second conjecture, the valley structure implies the existence of the phase transition. Note that the valley locates at low  $T < 20$  MeV and at high  $\mu \sim 950$  MeV, which indeed coincides with the region of the nuclear *liquid-gas phase transition*. As temperature is increased, a critical line separating a nucleon gas phase and a nuclear matter (liquid) disappears at around  $T \sim 15$  MeV, and above that temperature, there is no distinction between a gas and a liquid. This seems to be consistent with the disappearance of valley structure with increasing temperature as shown in the right panel of Fig. 2. The similar conclusion was obtained from the results of low energy effective theories [5].

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